Mixed Phase in Compact Stars : M-R relations and Radial oscillations

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It is believed that quark stars or neutron stars with mixed phase in the core have smaller radii compared to ordinary compact stars. With the recent observation of several low radius objects, typically a radius of $< 10Km.$ for star of mass $< 1M_0$ in low mass X-ray binaries (LMXB) , it has become very important to understand the nature of these objects. An accurate determination of mass-radius relationship of these objects provide us with a physical laboratory to study the composition of high density matter and the nature of phase transition. We study the effect of quark and nuclear matter mixed phase on mass radius relationship and radial oscillations of neutron stars. We find that the effect of the mixed phase is to decrease the maximum mass of a stable neutron star and to decrease the radial frequencies .

1. Introduction

Neutron stars are one of the most fascinating compact objects in the Universe coming into existence at the end of the evolutionary journey of a massive star. They support themselves against gravitational collapse by the pressure of degenerate neutrons and are the most compact and dense stars known. They typically compress a solar mass matter into a tiny radius of 10 Km with densities in the core reaching several times the nuclear density. They thus provide a unique opportunity to study properties of matter, its composition and phases, at extremely high densities and to perform tests of General theory of relativity (GTR). With such densities in the core, they themselves can take various forms, for example they could be composed of normal nuclear matter with hyperons and/or condensed mesons. The matter at such densities may undergo phase transition to constituent quark matter. It would then be energetically profitable for the u-d matter to convert itself into u-d-s matter through weak interactions thereby lowering its energy per baryon. If it so happens that the energy per baryon of such matter called **strange matter** (SQM) *i.e.,* matter with roughly equal number of u, d and s quarks with electrons to guarntee charge neutrality, is the true ground state of matter with energy per baryon less than that of iron (939 MeV) the most stable nuclei, the whole star will convert itself into what is called a **strange star** with vastly different characteristics. Further, since at large densities quantum chromodynamics (QCD) becomes a weakly interacting

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theory of quarks and gluons, the attractive force between quarks near the Fermi surface due to one gluon exchange will result in the formation of Cooper pairs giving rise to a **Color Superconducting State** wherein the Cooper pairs will condense breaking the color gauge symmetry [1]. In general QCD at high densities has a very rich phase structure, but the pairing pattern formed at sufficiently high densities for massless quarks is the **color flavored locked (CFL)** phase in which quarks of all three colors and flavors are paired in a single condensate. In this phase SQM is neutral and no electrons are present. If the mass of strange quark is not too large , this phase may extend to even low densities but for $m_s \sim 150MeV$, one gets a two flavor super conductor (2SC) phase in which only u and d quarks of two colors are paired to form a condensate and the third color pairs with strange quark. In case strange matter is not the true ground state, the neutron star may have a quark core followed by a mixed phase and nuclear mantle at the top. Such stars are called **hybrid stars** .

A neutron star comes into existence through a cataclysmic process where all the fundamental forces of nature come into play. The time scale is large compared to not only the strong and electromagnetic interactions time scale but weak interaction time scale as well. Further the electrostatic repulsion being so much stronger compared to gravitational attraction, the matter is electrically neutral and typical Fermi momenta of constituents being large compared to its temperature, neutron star is composed of **cold, degenerate, charge neutral matter in β-equilibrium**. At densities $< 2 \times 10^{-3}\rho_0$ ($\rho_0 = 0.16$ $nucleons\ /fm^3$ is the equilibrium density of charged nuclear matter in nuclei), the matter is assumed to be in the form of a Coulomb lattice (to minimize energy) of nuclei immersed in a relativistic degenerate electron gas. In the lower part of the density range $10^{-3}\rho_0 < \rho < 10\rho_0$ the neutrons leak out of the nuclei and a Fermi liquid of neutrons, protons and electrons start building up. At higher densities there are several possibilities including the occurance of muons, condensation of negatively charged pions and kaons, appearance of hyperons and finally, the transition to quark matter.

Matter at such high densities has not been produced in the laboratory and there is no available data on nuclear matter interactions. The quantities of interest are the phases and composition of neutron star matter, its energy density and pressure which determine the equation of state (EOS). The EOS upto nuclear densities is fairly well accounted for on the basis of measured nuclear data and nucleonic interactions. Above nuclear densities there are basically two approaches: one is to take the interaction between constituents from realistic fitting with known scattering data and then use the techniques of many body theory to calculate correlations. The other is to take a relativistic mean field type of model with couplings treated as parameters to fit observable quantities. Both approaches suffer from lack of experimental data. Whereas many body approach is well understood, two nucleon interactions are fairly well known, higher body interactions are not well characterised and the approach is non-relativistic. Mean field theoretical models can easily incorporate many constituents, but the complicated correlations are simplified in terms

of vacuum expectation of mean fields which are fitted to insufficient data.

As discussed above, if there is a phase transition to quark matter, the entire star may convert itself into a strange star or a hybrid star depending on whether the strange matter is the true ground state of matter or not. This has indeed shown to be possible in the MIT bag model with realistic values of the parameter wherein the long range cofining QCD interactions are taken into account phenomenologically by bag pressure and short range interactions perturbatively. Recently a number of other models[2] such as the Effective Mass Bag Model , density dependent quark mass model , a model by Dey et al. in which the SQM has asymptotic freedom at high densities and confinement at zero density built in have been considered. In Dey's model, the quark interaction is described by a color-Debye-screened interquark vector potential arising from gluon exchange and density dependent scalar potential which restores chiral symmetry at high densities. In the event of strange matter not being the absolute ground state, as happens, for example, in the Nambu-Jona-Lasinio model for parameters fitted from low energy spectroscopy, or in the MIT model for different values of the parameters, the quark matter in normal phase or in the color superconducting phase as discussed above can exist in the inner regions of more massive stars or in the mixed phase in equilibrium with the confined hadronic phase or both. In earlier studies the phase tramsition was characterised as a first order transition with a single component viz baryon number and charge neutrality was strictly enforced in each phase separately. This gave rise to constant pressure (liquid-vapour) type phase transition and since in a star, the pressure increases monotonically with density as we go from the surface to the core, mixed phase was strictly prohibited. It was pointed out by Glendenning [3] that matter in neutron star has two components , namely the conserved baryon number and the electric charge, therefore the correct application of Gibb's phase rule is that the chemical potential corresponding to baryon number and charge conservation ie. μ_B and μ_Q , the temperature and the pressure in two phases are equal *i.e.*,

$$\mu_B(h) = \mu_B(q) \qquad ; \qquad \mu_Q(h) = \mu_Q(q)$$

$$p_h(\mu_B, \mu_Q, T) = p_q(\mu_B, \mu_Q, T)$$

and charge neutrality only demands Global conservation

$$\chi Q_q(\mu_B, \mu_Q, T) + (1 - \chi) Q_h(\mu_B, \mu_Q, T) = 0$$

where χ is the fraction of the volume occupied by the quark phase. The freedom available to the system to rearrange concentration of charges for a given fraction of phases χ, results in variation of the pressure through the mixed phase.

2. Radial Oscillations and Mass Radius

The equations governing the radial oscillations of a non-rotating star, using static, spherically symmetric metric

$$ds^2 = -e^{2\nu} dt^2 + e^{2\lambda} dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2) \tag{1}$$

were given by Chandrasekhar [4]. The structure of the star in hydrostatic equilibrium is determined by the Tolman-Openheimer-Volkoff equations

$$\frac{dp}{dr} = \frac{-G(p+\rho)(m+4\pi r^3 p)}{r^2(1-\frac{2GM}{r})} \tag{2}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho \tag{3}$$

$$\frac{d\nu}{dr} = \frac{2GM(1+\frac{4\pi r^3 p}{m})}{r(1-\frac{2GM}{r})} \tag{4}$$

where we have put $c = 1$. Assuming a radial displacement δr with harmonic time dependence $\delta r \sim e^{i\omega t}$ and defining variables $\xi = \frac{\delta r}{r}$ and $\zeta = r^2 e^{-\nu}\xi$, the equation governing radial adiabatic oscillations is given by

$$F\frac{d^2\zeta}{dr^2} + G\frac{d\zeta}{dr} + H\zeta = \omega^2\zeta \tag{5}$$

where

$$F = -\frac{e^{2\nu-2\lambda}(\gamma p)}{p+\rho} \tag{6}$$

$$G = -\frac{e^{2\nu-2\lambda}}{p+\rho}\left[\gamma p(\lambda+3\nu) + \frac{d(\gamma p)}{dr} - \frac{2}{r}(\gamma p)\right] \tag{7}$$

$$H = \frac{e^{2\nu-2\lambda}}{p+\rho}\left[\frac{4}{r}\frac{dp}{dr} + 8\pi G e^{2\lambda}p(p+\rho) - \frac{1}{p+\rho}(\frac{dp}{dr})^2\right] \tag{8}$$

λ is related to the metric function through

$$e^{-2\lambda} = (1 - \frac{2GM(r)}{r}) \tag{9}$$

and γ is the adiabatic index, related to the speed of sound through

$$\gamma = \frac{p+\rho}{p}\frac{dp}{d\rho} \tag{10}$$

Equation(5) is solved under the boundary conditions

$$\zeta(r=0) = 0 \qquad\qquad \delta p(r=R) = 0 \tag{11}$$

where $\delta p(r)$ is given by

$$\delta p(r) = -\frac{dp}{dr}\frac{e^{\nu}\zeta}{r^2} - \frac{\gamma p e^{\nu}}{r^2}\frac{d\zeta}{dr} \tag{12}$$

Equation (5) with the boundary condition (11) represent a Sturm-Liouville eigenvalue problem for ω^2 with the well known result that the frequency spectrum is descrete. For $\omega^2 > 0$, ω is real and the solution is purely oscillatory whereas for $\omega^2 < 0$, ω is imaginery resulting in exponentially growing unstable radial oscillations. Another important consequence is that if the fundamental radial mode ω_0 is

stable, so are the rest of the radial modes. For neutron stars ω_0 becomes imaginery at central densities $\rho_c > \rho_c^{critical}$ for which the star attains it's maximum mass. For $\rho_c = \rho_c^{critical}$, the fundamental frequency ω_0 vanishes and becomes unstable for higher densities and the star is no longer stable. There also exists another unstable point at the lower end of the central density, namely there exists a minimum mass for a stable neutron star and the frequency of the fundamental mode at the minimum mass again goes to zero.

The only information required to obtain the structure of the star and eigenvalues of radial oscillation modes is the knowledge of the EOS. For a given EOS, equations (2)-(5) are solved numerically by standard techniques under the boundary conditions (11). While numerically integrating the equations for each EOS we make sure that the eigenfrequency of the fundamental mode goes to zero at the maximum mass of the star. We have also checked that the frequency vanishes at the minimum stable mass too. For nuclear matter EOS we use two relativistic mean field theoretic models taken from Glendenning [5] and two potential models incorporating relativistic corrections and three body interactions given by Akaml, Pandharipande and Ravenhall [6]. Both class of models admit of mixed quark-nucleon phase in the core. The quark matter in the above EOS models is described by the MIT Bag Model with $m_u = m_d = 0$, $m_s = 150$ MeV, the Bag constant $B^{\frac{1}{4}} = 180$ MeV and $\alpha_s = 0$. To illustrate the effect of mixed phase on neutron star parameters, namely

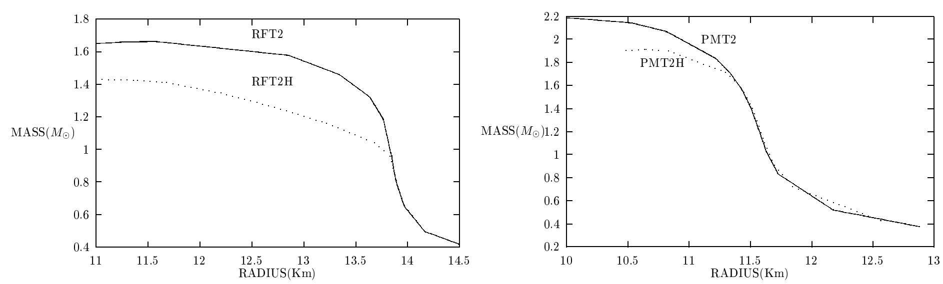

Fig. 1. Plot of mass in solar mass units versus radius in Km. Graphs labbeled as in reference 7.

mass-radius relationship and on the frequency of radial oscillations, in Fig.1 we have plotted [7] the M-R curves and find that the effect of the existence of mixed quark-nuclear matter phase in the core of neutron stars is to reduce the maximum mass. The effect is more pronounced for the relativistic mean field theory model than for the potential model. In Fig.2 we have plotted the frequency of the fundamental mode and the next mode as a function of central density for the pure neutron and hybrid stars in the two categories and find that the frequency exhibits oscillatory behaviour in the case of a neutron star with a mixed quark-nuclear matter core.

The mass of a strange star in contrast to neutron star on the other hand varies as $\sim R^3$ for $M \sim 0.5 M_0$ and gravity plays minimal role, bag pressure provides the confinement and the star is self bound. As mass increases gravity becomes

important and the star reaches a maximum mass. In contrast to neutron stars whose radii increase with decreasing mass and there is a minimum mass, strange star radii decreas with decrease in mass and there is no minimum mass. Strange star density falls abruptly to $\rho = 4B$ at the surface whereas in the the neutron star case it falls to zero at the surface. The pure quark stars as expected are more compact and they have much smaller radii with slightly smaller value of maximum mass for the casewhen strange matter is modeled by the EOS given by Dey et al. and for the CFL and superconducting phase. In the MIT bag model, there is little difference in M-R of a 1.4 M_0 neutron or strange star. The effect of CFL/superconducting phase in hybrid stars is similar to that shown in Figs.1 & 2. The radial oscillation modes too exibit behaviour similar to that of neutron stars with similar values for the frequency [8].
Thus to summarize

- The potential models give larger maximum stable mass upto 2.2 M_0 and higher values of radial frequencies in contrast to relativistic mean field models.
- The effect of the mixed phase is to soften the EOS thereby lowering the maximum mass as well as the radial frequencies to the extent of $\sim 30\%$.
- For a 1.4 M_0 star, there is a substantial change in radial frequency for the RFT models and none for PMT models.
- For a strange star, the radius decreases with mass and can be substantially smaller in comparison to a neutron star radius and the radial frequencies are of the same order as for neutron stars.

3. Observations and Conclusions

During the last thirty years since the discovery of first pulsar in 1968 by Hewish et.al. close to two thousand pulsars using radio telescope and X-ray probes in space have been discovered in a variety of circumstances;

- as isolated radio sources at times in binary orbits with other stars
- as X-ray pulsars and X-ray bursters in X-ray binary system
- most recently by Rossi XTE X-ray satellite as KHz quasi periodic oscillations (QPO) and burst oscillations in LMXB's.

A careful determination of the mass-radius relationship of these objects has led not only to their identification with neutron stars but has provided physical laboratory with unprecedented potentialities to perform tests of GTR and to obtain information on the EOS of high density matter its composition and phase transition.The most accurate determination of neutron star masses is found in binary pulsars and the masses of all these neutron stars lie in the range $1.35 + 0.04M_0$ [9] with exceptions of PSR J1012 of mass $2.1 + 0.4M_0$. Masses of X-ray pulsars are measured less accurately and recent observations for Vela X-1 and Cygnus X-2 give $1.9 + 0.2M_0$ and $1.8 + 0.4M_0$ respectively [10]. The recent discoveries of kilohertz-quasi-periodic

oscillations in LMXB's provide a new method for determining masses and radii of neutron stars from the detection of X-ray pulsations by requiring that the inner radius of accretion flow R_0 be less than the radius of the star R but less than the corotation radius R_c so that accretion is not centrifugally inhibited ie. $R < R_0 < R_c$. Based on these considerations a mass-radius relationship $R < 8.54(\frac{M}{M_0})^{1/2}$Km for SAX J1808.4-3658 has been obtained [11]. Stringent constraints on the M-R relation [12] have also been obtained for the X-ray sources 4U 1728-34 ($M < 1.0M_0$ and $R < 9$Km), for isolated compact star RX J1856-37 (M=0.9$\pm$0.2M_0, R=6+2-3 Km), for X-ray pulsar Her X-1 (M=1.1-1.8 M_o, R=6-7.7 Km). Most recently for the case of isolated neutron star RXJ1856 a radius $<$ 6Km has been obtained from the Chandra and XMM measurments of X-ray spectra [13]. Thus

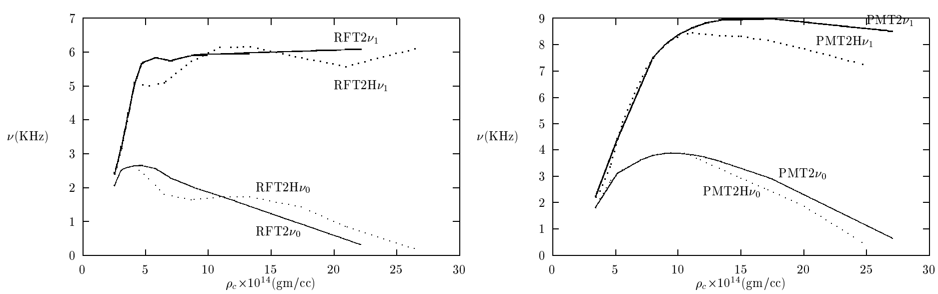

Fig. 2. Plot of frequency in KHz Vs central density in gm/cc

- If large masses of neutron stars are confirmed and complemented by other neutron star masses $\sim 2M_0$, EOS is severely restricted and only stiff EOS's without any significant phase transition below $5n_0$ are allowed. In this scenario existence of strange stars is not required.
- On the other hand if heavy neutron stars prove erroneous by more detailed observations and masses like those of binary pulsars ($\sim 1.4M_0$) alone are found , this will indicate that accretion does not produce heavier stars which will mean either a soft EOS or significant phase transition at few times the nuclear saturation density.
- Observations of stars of mass $\leq M_0$ and radius $\leq 10Km$ as seems to be borne out by present analysis of X-ray sources albeit with uncertainities in our knowledge of accretion mechanism and realistic EOS for the quark matter would imply the possibility of their identification with strange stars.

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